

**NASA TECHNICAL  
MEMORANDUM**

**NASA TM X-71810**

**NASA TM X-71810**

(NASA-TM-X-71810) ANALYSIS AND CORRECTION  
OF GROUND REFLECTION EFFECTS IN MEASURED  
NARROWBAND SOUND SPECTRA USING CEPSTRAL  
TECHNIQUES (NASA) 17 p HC \$3.25 CSCL 20A

**N76-10820**

**Unclassified**

**G3/71 03055**

**ANALYSIS AND CORRECTION OF GROUND REFLECTION EFFECTS IN  
MEASURED NARROWBAND SOUND SPECTRA  
USING CEPSTRAL TECHNIQUES**

by J. H. Miles, G. H. Stevens, and G. G. Leininger  
Lewis Research Center  
Cleveland, Ohio 44135

**PRICES SUBJECT TO GOVERNMENT CONTRACT**

**TECHNICAL PAPER to be presented at Ninetieth Meeting of the  
Acoustical Society of America  
San Francisco, California, November 4-7, 1975**

Reproduced by  
**NATIONAL TECHNICAL  
INFORMATION SERVICE**  
U.S. Department of Commerce  
Springfield, VA. 22151

ANALYSIS AND CORRECTION OF GROUND REFLECTION EFFECTS IN MEASURED  
NARROWBAND SOUND SPECTRA USING CEPSTRAL TECHNIQUES  
by J. H. Miles,<sup>\*</sup> G. H. Stevens,<sup>\*</sup> and G. G. Leininger<sup>†</sup>

ABSTRACT

Ground reflections generate undesirable effects on acoustic measurements such as those conducted outdoors for jet noise research, aircraft certification, and motor vehicle regulation. This paper shows how cepstral techniques developed in speech processing can be adapted to identify the echo delay time and to correct for ground reflection effects. A sample result is presented using an actual narrowband sound pressure level spectrum. The technique can readily be adapted to existing fast Fourier transform type spectrum measurement instrumentation to provide field measurement of echo time delays.

E-8503

LIST OF SYMBOLS

$a_0$	amplitude parameter
$b_n$	constant
$C(q)$	cepstrum
$F[]$	Fourier transform operator
$F^{-1}[]$	inverse Fourier transform
$f$	frequency
$H(\omega)$	received signal power spectral density
$i$	index valve for discrete quantities
$j$	$(-1)^{1/2}$
$k$	integer constant
$L_c$	cepstrum level

---

<sup>\*</sup>NASA Lewis Research Center.

<sup>†</sup>Electrical Engineering Dept., Univ. of Toledo, Toledo, Ohio 43606.

n	integer constant
P( $\omega$ )	direct signal power spectral density
Q( $\omega$ )	amplitude of the complex reflected signal transfer function
q	quefrency
SPL( $\omega$ )	sound pressure level re $2 \times 10^{-5}$ Pa, dB
$ T(\omega) ^2$	reflectance
y	constant
z	ratio of path length of the reflected signal to the path length of the direct signal
$\alpha$	phase angle parameter, $\omega\tau - \theta(\omega)$
$\delta$	delta function
$\theta(\omega)$	phase angle of the complex reflected transfer function
$\tau$	time delay
$\omega$	angular frequency, $2\pi f$

#### Subscripts:

m	measured
$\Sigma$	intensity - summed

#### Superscript:

+	without echo effects
---	----------------------

## INTRODUCTION

The fundamental problem discussed herein is to identify and remove ground reflection effects so that the true narrowband spectral emission properties of a noise source may be determined. The particular problem discussed commonly occurs when noise measurements of predominantly low frequency noise sources such as full scale jets, large-scale blown flaps or combustors are made over a ground plane. Due to NASA activity in air-

craft noise reduction, considerable effort at NASA Lewis Research Center has gone into experimental and theoretical investigations aimed at minimizing reflection effects.<sup>1-4</sup> Ground reflection effects are also important because they may cause spectrum measurement errors that can have a large economic impact due to various federal, state, and city noise control regulations.<sup>5,6</sup>

An analytical solution to the problem for plane waves is simple in principle. However, real effects such as distributed sources, nonplanar waves, temperature and wind-created shadow zones complicate the problem and render analytical solutions difficult. Thus, emphasis has been placed on experimental procedures to eliminate ground reflection effects such as using ground microphones or overhead-boom microphones (which can be subject to operational problems), and computer model-inverse filter type procedures to correct ground reflection effects in measured data (which are partially empirical).<sup>2-4</sup>

A new procedure to identify and remove reflection effects is presented herein which avoids previous problems. The technique described draws upon an analytical method of signal processing known as cepstrum analysis. In the present work, the cepstrum is the Fourier transform of the sound pressure level spectrum which has the necessary property of being a logarithmic function of the sound pressure. The cepstrum technique has been used for echo detection,<sup>7-12</sup> speech analysis (i.e., finding the pitch period of a voiced speech segment),<sup>13,14</sup> and photographic processing.<sup>15,16</sup> This paper discusses how the cepstrum technique was adapted to identifying the echo time delays on the ground reflection problem. The method is sufficiently general to eliminate phase cancel-

lation and reinforcement effects due to multiple reflecting surfaces and produces a spectrum resulting from only intensity addition of the direct and reflected signals.

This paper contains first a discussion of the cepstral technique as applied to the analysis of ground reflection effected sound pressure level spectral data. The calculation method is next discussed, and an example is presented using an actual experimental sound pressure level spectrum. Finally, some remarks on possible application to real time data analysis are given.

#### THEORY

##### Effect of Reflections on the Spectrum

The procedure described herein for identifying and removing reflection effects is based on the effect reflections have on the spectrum. For a signal having a deterministic waveform or a signal from a real random ergodic process, the effect is the same. In Ref. 3, this effect is derived for a multiple noise source treated as a real random ergodic process. The resulting equation shows that the power spectral density of the received signal  $H(\omega)$  is the product of the spectral density of the direct signal  $P(\omega)$  and a transfer function corresponding to the reflection process called the reflectance  $|T(\omega)|^2$ . Thus,

$$H(\omega) = P(\omega) |T(\omega)|^2 \quad (1)$$

The reflectance for a single point source is given by

$$|T(\omega)|^2 = 1 + \frac{Q^2(\omega)}{z^2} + \frac{2Q(\omega)}{z} \cos[\omega\tau - \theta(\omega)] \quad (2)$$

where  $\omega$  is the angular frequency,  $Q(\omega)$  is the amplitude, and  $\theta(\omega)$  is the phase angle of the complex reflected signal transfer function (be-

cause amplification does not occur along the reflected signal path  $Q(\omega)$  has a maximum value of 1),  $\tau$  is the echo time delay from source to microphone and  $z$  is the ratio of the path length of the reflected signal to the path length of the direct signal.

The reflectance for a point source (Eq. 2) and the more general reflectance derived in Ref. 3 for a distributed source have a cosinusoidal form. This functional form is the basis of the procedure developed to correct the measured spectrum. The procedure will be discussed next.

#### The Cepstrum

As indicated, the power spectral density of the composite signal (consisting of the direct and reflected signal) is the product of two terms. The first term ( $P(\omega)$ ) is the power spectral density of the direct signal. The second term ( $|T(\omega)|^2$ ), due to the echo, is periodic and produces a nearly cosinusoidal ripple in the spectrum of the composite signal. This ripple is readily observable in the sound pressure level spectrum which is defined as ten times the logarithm of the power spectral density of the received signal. From Eq. 1 the received spectrum is written as

$$\text{SPL}(\omega) = 10 \log_{10} H(\omega) = 10 \log_{10} P(\omega) + 10 \log_{10} |T(\omega)|^2 \quad (3)$$

Substitution of Eq. 2 into Eq. 3 produces the expression

$$\text{SPL}(\omega) = 10 \log_{10} P(\omega) + 10 \log_{10} \left\{ 1 + \frac{Q^2(\omega)}{z^2} + \frac{2Q(\omega)}{z} \cos[\omega\tau - \theta(\omega)] \right\} \quad (4)$$

The effect of reflection on the sound pressure level spectrum is periodic. From Eq. 4, the reflectance term has spectral peaks at frequencies which are integral multiples of  $1/\tau$  plus a phase factor  $\theta(\omega)/2\pi\tau$ .

The ripple observed in the spectrum received by a microphone near the ground is from Eq. 4 periodic in frequency. For functions periodic in time, the reciprocal of the period (time spacing between cycles) is frequency and has dimensions of cycles per second. By convention, to prevent any confusion with this latter terminology, the reciprocal of the period of the spectral ripple is called the quefrency<sup>13,14</sup> and has the dimension of time. Thus, short period ripple in a sound spectrum corresponds to high quefrency components. The observed spectrum received by a microphone near the ground generally consists then of high quefrency components caused by reflections superimposed on the direct sound spectrum.

Just as the Fourier transform of a signal can be used to identify frequency components of a signal, the Fourier transform of the sound pressure level spectrum can be used to identify quefrency components in the spectrum. Such an analysis produces the cepstrum. While frequency is the independent variable in the spectrum, quefrency,  $q$ , is the independent variable in the cepstrum. For noise data measured near a ground plane, the cepstrum contains large peaks at the quefrequencies related to echo delay times and multiples thereof. Thus, cepstral analysis is an effective way to find echo delay times.

This can be shown as follows. First, Eq. 4 is written as

$$\text{SPL}(\omega) = \text{SPL}^+(\omega) + 10 \log_{10} \left[ 1 + \frac{Q^2(\omega)}{z^2} \right] + 10 \log_{10} \left\{ 1 + \frac{2Q(\omega)}{z \left[ 1 + \frac{Q^2(\omega)}{z^2} \right]} \cos[\omega\tau - \theta(\omega)] \right\} \quad (5)$$

where

$$\text{SPL}^+(\omega) = 10 \log_{10} P(\omega) \quad (6)$$

Now, let

$$a_0 = \frac{2Q(\omega)}{z \left[ 1 + \frac{Q^2(\omega)}{z^2} \right]} \quad (7)$$

and

$$\alpha = \omega T - \theta(\omega) \quad (8)$$

From the definition of  $Q(\omega)$  and  $z$ , it follows that  $a_0$  is less than or equal to one. Thus, the third term of Eq. 5 can be expanded by the series expression for the Napierian logarithm for  $(1 + y)$

$$\ln(1 + y) = \sum_{n=0}^{\infty} (-1)^{n-1} y^n / n \quad (9)$$

which converges provided

$$-1 < y \leq 1$$

Substitution of Eqs. 7 and 8 into Eq. 5 and using Eq. 9 and the relation of logarithms of base 10 with logarithms of base e permits the third term of Eq. 5 to be expressed as follows:

$$10 \log_{10}(1 + a_0 \cos \alpha) = 10 \log_{10} e \sum_{n=0}^{\infty} (-1)^{n-1} a_0^n \frac{\cos^n \alpha}{n} \quad (10)$$

Powers of  $\cos \alpha$  can be expressed as

$$\cos^n \alpha = \left( \frac{e^{j\alpha} + e^{-j\alpha}}{2} \right)^n \quad (11)$$

Using this relation in Eq. 10 and substituting the result in Eq. 5 shows that

$$SPL(\omega) = SPL^+(\omega) + 10 \log_{10} \left[ 1 + \frac{Q^2(\omega)}{z^2} \right] + \sum_{n=0}^{\infty} b_n \cos \{ n[\omega\tau - \theta(\omega)] \} \quad (12)$$

The Fourier transform of Eq. 12 is formed to obtain the cepstrum  $C(q)$ . For the case where  $\theta(\omega)$  is a constant times  $\omega$  and  $a_0$  is independent of  $\omega$ ,

$$\begin{aligned} C(q) &= F[SPL(\omega)] = F[SPL^+(\omega)] \\ &+ F \left[ 10 \log_{10} \left( 1 + \frac{Q^2(\omega)}{z^2} \right) \right] + \sum_{n=0}^{\infty} b_n \delta \left\{ q - n \left[ \tau - \frac{\theta(\omega)}{\omega} \right] \right\} \end{aligned} \quad (13)$$

Therefore the cepstrum for the single echo case consists of the cepstrum of the "intensity-summed" spectrum plus a train of impulses (i.e., delta functions) occurring at times which are integral multiples of  $\tau - \theta(\omega)/\omega$ . Intensity summing implies that the combined energy of the direct and reflected signal is obtained and that reinforcements and cancellations due to signal phasing are absent. The intensity-summed cepstrum  $C_{\Sigma}^+(q)$  is therefore

$$C_{\Sigma}^+(q) = F[SPL^+(\omega)] + F \left\{ 10 \log_{10} \left[ 1 + \frac{Q^2(\omega)}{z^2} \right] \right\} \quad (14)$$

#### Intensity-Summed Spectrum Extraction Procedure

The intensity-summed spectrum is extracted from the cepstrum by first smoothing the cepstrum. This is done to remove the delta functions due to the echoes and may be accomplished by interpolating between points adjacent to the peaks which appear in the cepstrum due to the echo. The resulting intensity-summed cepstrum is then inverse-Fourier-transformed. From Eq. 14 the resulting intensity-summed sound pressure level spectrum is

$$\text{SPL}_{\Sigma}^+(\omega) = F^{-1}[C_{\Sigma}^+(q)] = \text{SPL}^+(\omega) + 10 \log_{10} \left[ 1 + \frac{Q^2(\omega)}{z^2} \right] \quad (15)$$

Thus, all phase cancellations and reinforcements due to echoes have been removed.

#### COMPUTATION METHOD

This section describes the computational procedure used. The following section discusses results obtained with real data.

##### Fourier Transform Computation

The Fourier transform used to compute the cepstrum was performed using the fast Fourier transform method of computing the discrete Fourier transform. (Fast Fourier transform methods of analyzing sampled time series data are described in Refs. 17 to 23.) The Fortran computer program used herein to implement the fast Fourier transform is described in Ref. 20 and requires  $2^k$  points, where  $k$  is some integer.

##### Cepstrum Smoothing Procedures

Methods for smoothing the cepstrum are discussed in Ref. 8. The basic procedure is designed to remove the periodic ripple in the spectrum introduced by the multiple echoes. This can be achieved by multiplying the cepstrum by an appropriate function. The method used herein is called "comb filtering."

Comb filtering is performed by multiplying the cepstrum by a function that is unity everywhere except at those points which are considered to be due to echoes. At these points the filter function is zero. The resulting filter cepstrum is then found at the zero points by linearly interpolating between the preceding and subsequent nonzero points in the cepstrum.

Selection of the cepstrum quefrequencies due to echoes must be done with caution since not all nonzero values of the cepstrum are due necessarily to echo time delays. For example, the cepstrum of physically separated but coherent sources can resemble one with echoes. Also, a double peaked spectrum (typical of a suppressor nozzle) will introduce nonzero components in the cepstrum which as previously discussed are subject to misinterpretation. The proper quefrency selection can be achieved in a number of ways. With the above cautions in mind the simplest procedure is to assume that all spikes larger than a given value at quefrequencies larger than the lowest reasonable echo delay time are actually due to echoes.

The steps followed by the method are as follows:

- (1) Form the new sound pressure level spectrum array;
- (2) Calculate the transform to produce  $C(q)$ ;
- (3) Apply the comb filter to produce  $C_{\Sigma}^+(q)$ .
- (4) Inverse transform the cepstrum to produce  $SPL_{\Sigma}^+(\omega)$ .

This is schematically diagrammed in Fig. 1.

## RESULTS AND DISCUSSION

### Application to Measured Data

Spectrum. - To test the cepstrum procedure on real measured data a set of graphical narrowband jet noise data was selected. The data are, subsonic hot jet noise data from a 10.16 cm nozzle operated with a velocity of 548.64 m/sec at a temperature of 649° C (1200° F) taken over an asphalt surface with geometry as follows: microphone height, 1.676 m; source-microphone distance, 7.620 m; source height, 1.67 m; ratio of echo path length to direct signal path length, 1.09252 m. One hundred points

were read from the narrowband plot. For the cepstrum procedure, 128 data points ( $k = 7$ ) were needed and they were obtained by linearly interpolating 128 points from the 100 measured points. The first 100 interpolated points are shown in Fig. 2. The ripple due to ground reflection effects is very pronounced below 4700 Hz. Beyond 4700 Hz the ripple effect almost disappears.

Cepstrum. - The cepstrum was calculated using the method described above. Rather than showing the resulting cepstrum which has an approximate 4 decade range, the cepstrum level given by

$$L_c = 10 \log |C(q)|^2 \quad (17)$$

is presented in Fig. 3. Most of the energy, as discussed earlier, is at a quefrency of zero which corresponds to the energy associated with the direct signal. However, significant energy is also apparent at quefrequencies between  $0.2 \times 10^{-2}$  and  $0.205 \times 10^{-2}$  seconds and again between  $0.405 \times 10^{-2}$  and  $0.410 \times 10^{-2}$  seconds. Thus the echo time delay corresponding to the large peak at a quefrency of  $0.205 \times 10^{-2}$  seconds is easily identified by the method.

Intensity-summed spectrum. - The comb filter was applied to the cepstrum. The points used in the comb filter are indicated by the tick marks in Fig. 3.

The resulting intensity-summed spectrum obtained by inverse transforming the comb filtered cepstrum is shown by the solid curve in Fig. 4. The ripple at low frequencies has been essentially removed.

#### Application to Real Time Data Analysis

The cepstrum method of analyzing sound pressure level spectra de-

scribed herein can be used to identify echo time delays and to remove reflection effects in measured spectra as the data are being taken. The example given herein used graphical data for convenience. Digital signal processing instrumentation using fast Fourier transform techniques could be modified to produce cepstrum level plots so that echo time delays could be identified in real time. The comb filtering process and inverse Fourier transform calculation could also be performed automatically so that intensity summed sound pressure level spectra could be calculated in real time.

#### CONCLUDING REMARKS

This paper has shown that the effect of ground reflections on narrowband jet noise spectrum data can be adequately removed by cepstral techniques. Cepstrum calculation is a signal processing method developed originally for speech analysis and used for echo detection and photographic processing. The method developed herein extends the use of cepstral techniques into the field of aeroacoustics.

When the method was applied to measured jet noise data, it was found to eliminate all significant phase cancellation and reinforcement effects due to reflecting surfaces, producing a spectrum due to intensity addition only of the direct and reflected signals.

The method could be adapted to current instrumentation to provide a direct measurement of time delays due to reflection effects and thereby identify sources of echoes. This would be useful for aircraft noise certification, motor vehicle noise regulation, and jet noise research acoustic testing.

## REFERENCES

1. W. L. Newes, "Ground Reflection of Jet Noise," NASA TR R-35 (1959)
2. J. H. Miles, "Rational Function Representation of Flap Noise Spectra Including Correction for Reflection Effects," AIAA Paper 74-193 (1974)
3. J. H. Miles, "Method of Representation of Acoustic Spectra and Reflection Corrections Applied to Externally Blown Flap Noise," NASA TM X-3179 (1975).
4. J. H. Miles, "Analysis of Ground Reflection of Jet Noise Obtained with Various Microphone Arrays Over an Asphalt Surface," presented at the Eighty-Ninth Meeting of the Acoust Soc Am, Austin, Texas (April 1975).
5. N. Shapiro and J. W. Vogel, "Noise Certification of a Transport Airplane," in Inter-Noise 72; International Conference on Noise Control Engineering, Proceedings (Inst. Noise Control Eng.), 338-343 (1972)
6. B. H. Sharp, "The Measurement of Noise From Motor Vehicles on the Highway," in Inter-Noise 72; International Conference on Noise Control Engineering, Proceedings (Inst. Noise Control Eng.), 220-224 (1972).
7. B. P. Bogert and J. P. Ossana, "The Heuristics of Cepstrum Analysis of a Stationary Complex Echoed Gaussian Signal in Stationary Gaussian Noise," IEEE Trans. Inform Theory, IT-12, 373-380 (1966)
8. D. G. Childers and R. S. Varga, "Composite Signal Decomposition," IEEE Trans. on Audio and Electroacoustics, AU-18, 471-477 (1960).
9. R. C. Remerait and D. G. Childers, "Composite Signal Decomposition by Cepstrum Techniques," IEEE Region 3 Convention Southeastcon 1972, Proc. of 10th Annual: Scanning the Spectrum (Inst. Electrical and Electronics Engrs.) (1972).

10. S. Sennoto and D. G. Childers, "Adaptive Decomposition of a Composite Signal of Identical Unknown Wavelets in Noise," IEEE Trans. on Systems, Man and Cybernetics, SMC-2, 59-66 (1972).
11. R. C. Kemerait and D. G. Childers, "Signal Detection and Extraction by Cepstrum Techniques," IEEE Trans. on Inform. Theory, IT-18, 745-759 (1972).
12. R. G. Smith, "Cepstrum Discrimination Function," IEEE Trans. on Inform. Theory, IT-21, 332-334 (1975).
13. A. M. Noll, "Short-Time Spectrum and Cepstrum Techniques for Vocal-Pitch Detection," J. Acoust. Soc. Am., 36, 296-302 (1964).
14. A. M. Noll, "Cepstrum Pitch Determination," J. Acoust. Soc. Am., 41, 293-309 (1967).
15. A. V. Oppenheim, R. W. Schafer, and T. G. Stockman, Jr., "Nonlinear Filtering of Multiplied and Convolved Signals," Proc. IEEE, 56, 1264-1291 (1968).
16. R. Rom, "On the Cepstrum of Two-Dimensional Functions," IEEE Trans. on Inform. Theory, IT-21, 214-217 (1975).
17. J. W. Cooley, P. A. W. Lewis, and P. D. Welch, "Application of the Fast Fourier Transform to Computation of Fourier Integrals, Fourier Series, and Convolution Integrals," IEEE Trans. on Audio and Electroacoustics, AU-15, 79-84 (1967).
18. W. T. Cochran, et al., "What is the Fast Fourier Transform," IEEE Trans. on Audio and Electroacoustics, AU-16, 45-55 (1967).
19. G. D. Bergland, "A Guided Tour of the Fast Fourier Transform," IEEE Spectrum, 6, 41-52 (1969).

20. J. W. Cooley, P. A. W. Lewis, and P. D. Welch, "The Fast Fourier Transform and Its Applications," IEEE Trans. on Education, 12, 27-34 (1969).
21. J. W. Cooley, P. A. W. Lewis, and P. D. Welch, "The Finite Fourier Transform," IEEE Trans. on Audio and Electroacoustics, AU-17, 77-85 (1969).
22. S. Bertram, "On the Derivation of the Fast Fourier Transform," IEEE Trans. on Audio and Electroacoustics, AU-18, 55-58 (1970).
23. R. P. Brumbach, "Digital Computer Routines for Power Spectral Analysis," General Motors Corp. Tech. Rept. 68-31 (AD-673859) (Jul. 1968).

Pages 13, 14, BLANK

PRECEDING PAGE BLANK NOT FILMED

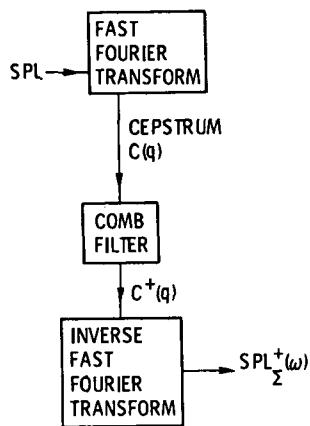


Figure 1. - Diagram of cepstrum analysis.

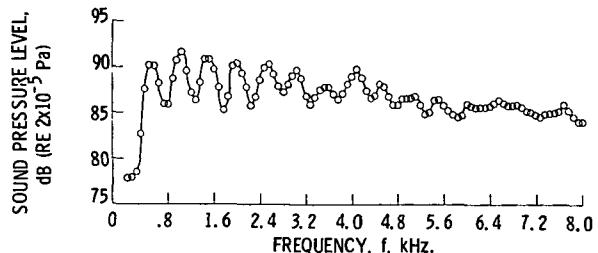


Figure 2. - Measured sound pressure level spectrum of jet noise.

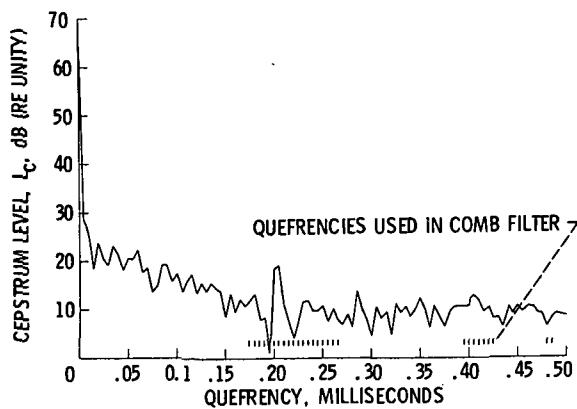


Figure 3. - Cepstrum level calculated from spectrum of figure 2.

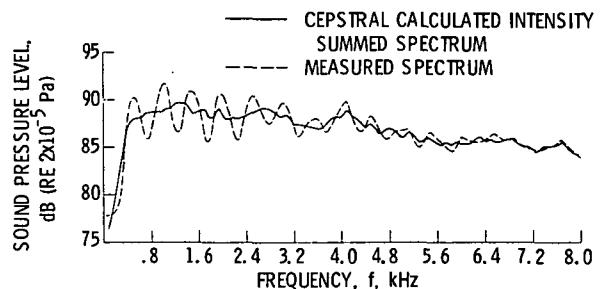


Figure 4. - Comparison of measured spectrum and cepstral calculated intensity summed spectrum.